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An explicit formula for the A -polynomial of the knot with Conway's notation $C(2n, 3)$

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ABSTRACT

An explicit formula for the A -polynomial of the knot with Conway's notation $C(2n, 3)$ is obtained from the explicit Riley-Mednykh polynomial of it.

Keywords: A -polynomial, explicit formula, knot with Conway's notation $C(2n, 3)$, Riley-Mednykh polynomial

Mathematics Subject Classification 2000: 57M27, 57M25

1. Introduction

In 1994, the A -polynomial, $A(L, M)$, of a compact 3-manifold N with a single torus boundary was introduced by Cooper, Culler, Gillet, Long, and Shalen in [3]. It's variables are eigenvalues of the meridian and the longitude under the representations from $\pi_1 N$ into $SL(2, C)$. One of the main results of [3] is "boundary slopes are boundary slopes". That is the boundary slope of the Newton polygon of $A(L, M)$ is the boundary slope of an incompressible surface in N . In 2001, it was shown that the Newton polygon of $A(L, M)$ is dual to the fundamental polygon of the Culler-Shalen seminorm [1]. The Culler-Shalen seminorm [4] can be used to detect and classify the exceptional surgeries, which is a step toward another proof of the Poincare conjecture. A -polynomial also encodes the deformed structure of N . For example, using the longitude L , in [11,9], the volumes of the deformed cone-manifolds are computed and in [8,10], the Chern-Simons invariants [2,12] of the deformed orbifolds are computed. The non-commutative A -polynomial $A(L, M, q)$ of a knot is introduced and it is conjectured that $A(L, M, 1) = B(M)A(L, M^{1/2})$

for some polynomial $B(M)$ of M and the conjecture is called AJ conjecture [6]. AJ conjecture is proved for some knots. For example, our knot, the knot with Conway's notation $C(2n, 3)$ satisfies the AJ conjecture [14]. If AJ conjecture is true then the colored Jones polynomial detects knottedness [5] as A -polynomial.

With today's technology, A -polynomial is relatively difficult to compute. Recovering representations from a triangulation of N and compute a factor of the A -polynomial is another try to compute it [20]. By one by one computation, A -polynomials are known up to 8 crossings and most 9 crossings and many 10 crossings. For infinite families, recursively, A -polynomials are known for twist knots [13], $(-2, 3, 1 + 2n)$ pretzel knots [19, 7], $J[m, 2n]$ [13] for m between 2 and 5 [17] and explicitly, for two-bridge torus knots [3, 13], iterated torus knots [16], and for twist knots [15, 8]. We record here that $J[3, -2n]$ is the mirror image of $C(2n, 3)$.

The main purpose of the paper is to find the explicit formula for the A -polynomial of the knot with Conway's notation $C(2n, 3)$. Let us denote the knot with Conway's notation $C(2n, 3)$ by T_{2n} and the A -polynomial of the knot with Conway's notation $C(2n, 3)$ by A_{2n} . The following theorem gives the explicit formula for the A -polynomial of T_{2n} .

Theorem 1.1. *A -polynomial $A_{2n} = A_{2n}(L, M)$ is given explicitly by*

$$A_{2n} = \begin{cases} \sum_{i=0}^{2n} \binom{n + \lfloor \frac{i}{2} \rfloor}{i} \left(\frac{(LM^{4n} - 1)(1 - M^2)}{1 + LM^{2+4n}} \right)^i \left(\frac{1 + LM^{6+4n}}{M^2 + LM^{4+4n}} \right)^{\lfloor \frac{1+i}{2} \rfloor} \\ \times M^{-2n} (1 + LM^{2+4n})^{3n} & \text{if } n \geq 0 \\ \sum_{i=0}^{-2n-1} \binom{-n + \lfloor \frac{i-1}{2} \rfloor}{i} \left(\frac{(1 - M^2)(M^{-4n} - L)}{LM^2 + M^{-4n}} \right)^i \left(\frac{LM^6 + M^{-4n}}{LM^4 + M^{2-4n}} \right)^{\lfloor \frac{1+i}{2} \rfloor} \\ \times M^{8n+6} (LM^2 + M^{-4n})^{-3n-1} & \text{if } n < 0 \end{cases}$$

One can consult [8] for solving the recurrence formula. Our writing is parallel with that in [15] which is based on [13].

2. Proof of Theorem 1.1

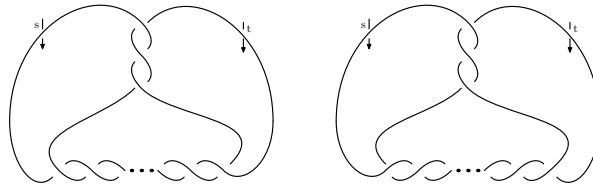


Fig. 1. A two bridge knot with Conway's notation $C(2n, 3)$ for $n > 0$ (left) and for $n < 0$ (right)

A knot K is a two bridge knot with Conway's notation $C(2n, 3)$ if K has a regular two-dimensional projection of the form in Figure 1. Recall that we denote it by T_{2n} . Let us denote the exterior of T_{2n} by X_{2n} . The following proposition gives the fundamental group of X_{2n} [9,10,13,18].

Proposition 2.1.

$$\pi_1(X_{2n}) = \langle s, t \mid swt^{-1}w^{-1} = 1 \rangle,$$

where $w = (ts^{-1}tst^{-1}s)^n$.

Given a set of generators, $\{s, t\}$, of the fundamental group for $\pi_1(X_{2n})$, we define a representation $\rho : \pi_1(X_{2n}) \rightarrow \text{SL}(2, \mathbb{C})$ by

$$\rho(s) = \begin{bmatrix} M & 1 \\ 0 & M^{-1} \end{bmatrix}, \quad \rho(t) = \begin{bmatrix} M & 0 \\ 2 - M^2 - M^{-2} - x & M^{-1} \end{bmatrix}.$$

Then ρ can be identified with the point $(M, x) \in \mathbb{C}^2$. When M varies we have an algebraic set whose defining equation is the following explicit Riley-Mednykh polynomial.

Lemma 2.2. ρ is a representation of $\pi_1(X_{2n})$ if and only if x is a root of the following Riley-Mednykh polynomial $P_{2n} = P_{2n}(x, M)$ which is given explicitly by

$$P_{2n} = \begin{cases} \sum_{i=0}^{2n} \binom{n+\lfloor \frac{i}{2} \rfloor}{i} M^{4n} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} & \text{if } n \geq 0, \\ \sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-2} (-M^2 - M^{-2} - x + 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} & \text{if } n < 0. \end{cases}$$

Proof. In [9], P_{2n} is give by the following recursive formula.

$$P_{2n} = \begin{cases} QP_{2(n-1)} - M^8 P_{2(n-2)} & \text{if } n > 1, \\ QP_{2(n+1)} - M^8 P_{2(n+2)} & \text{if } n < -1, \end{cases}$$

with initial conditions

$$\begin{aligned} P_{-2} &= M^2 x^2 + (M^4 - M^2 + 1)x + M^2, \\ P_0 &= M^{-2} \text{ for } n < 0 \quad \text{and} \quad P_0 = 1 \text{ for } n > 0, \\ P_2 &= -M^4 x^3 + (-2M^6 + M^4 - 2M^2)x^2 + (-M^8 + M^6 - 2M^4 + M^2 - 1)x + M^4, \end{aligned}$$

and

$$Q = -M^4 x^3 + (-2M^6 + 2M^4 - 2M^2)x^2 + (-M^8 + 2M^6 - 3M^4 + 2M^2 - 1)x + 2M^4.$$

We write f_{2n} for the claimed formula and show that $f_{2n} = P_{2n}$.

Case I: $n \geq 0$. When $i > 2n$ or $i < 0$, $\binom{n+\lfloor \frac{i}{2} \rfloor}{i}$ is undefined and can be considered as zero. Hence the finite sum can be regarded as an infinite sum. Direct computation shows that $f_0 = P_0$ and $f_2 = P_2$. Now, we only need to show that f_{2n} satisfies the recursive relation. Note that Q can be written as $M^4 \left(-x(M^2 + M^{-2} + x - 1)^2 + 2 \right)$.

$$\begin{aligned}
& Qf_{2(n-1)} - M^8 f_{2(n-2)} \\
&= M^4 \left(-x(M^2 + M^{-2} + x - 1)^2 + 2 \right) \\
&\times \sum_i \binom{n-1+\lfloor \frac{i}{2} \rfloor}{i} M^{4n-4} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
&- M^8 \sum_i \binom{n-2+\lfloor \frac{i}{2} \rfloor}{i} M^{4n-8} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
&= \sum_i \left[\binom{n-2+\lfloor \frac{i}{2} \rfloor}{i-2} + 2 \binom{n-1+\lfloor \frac{i}{2} \rfloor}{i} - \binom{n-2+\lfloor \frac{i}{2} \rfloor}{i} \right] \\
&\times M^{4n} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
&= \sum_i \binom{n+\lfloor \frac{i}{2} \rfloor}{i} M^{4n} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
&= f_{2n}
\end{aligned}$$

In the last equality we use the binomial relation

$$\binom{a}{b} = \binom{a-1}{b-1} + \binom{a-1}{b}$$

three times.

Case II: $n < 0$. When $i > -2n - 1$ or $i < 0$, $\binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i}$ is undefined and can be considered as zero. Hence the finite sum can be regarded as an infinite sum. Direct computation shows that $f_0 = P_0$ and $f_{-2} = P_{-2}$. Now, we only need to show that f_{2n} satisfies the recursive relation. We know that Q can be written as

$$\begin{aligned}
& M^4 \left(-x (M^2 + M^{-2} + x - 1)^2 + 2 \right). \\
& Qf_{2(n+1)} - M^8 f_{2(n+2)} \\
& = M^4 \left(-x (M^2 + M^{-2} + x - 1)^2 + 2 \right) \\
& \times \sum_i \binom{-n-1+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-6} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
& - M^8 \sum_i \binom{-n-2+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-10} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
& = \sum_i \left[\binom{-n-2+\lfloor \frac{i-1}{2} \rfloor}{i-2} + 2 \binom{-n-1+\lfloor \frac{i-1}{2} \rfloor}{i} - \binom{-n-2+\lfloor \frac{i-1}{2} \rfloor}{i} \right] \\
& \times M^{-4n-2} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
& = \sum_i \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-2} (M^2 + M^{-2} + x - 1)^i (-x)^{\lfloor \frac{1+i}{2} \rfloor} \\
& = f_{2n}
\end{aligned}$$

In the last equality we use the binomial relation

$$\binom{a}{b} = \binom{a-1}{b-1} + \binom{a-1}{b}$$

three times, again. \square

Let $l = ww^*s^{-4n}$ [3,13], where w^* is the word obtained by reversing w . Let $L = \rho(l)_{11}$. Then l is the longitude which is null-homologous in X_{2n} (you can read a twisted longitude ww^* from the Schubert normal form of the knot $C(2n, 3)$ and multiply it by s^{-4n} so that the exponent sum of l becomes 0). And we have

Lemma 2.3. [9,10]

$$\begin{aligned}
L &= -M^{-4n-2} \frac{M^{-2} + x}{M^2 + x}, \\
x &= -\frac{1 + LM^{6+4n}}{M^2(1 + LM^{2+4n})}.
\end{aligned}$$

Now substituting $-\frac{1+LM^{6+4n}}{M^2(1+LM^{2+4n})}$ for x into P_{2n} , for $n \geq 0$, gives

$$\begin{aligned}
& \sum_{i=0}^{2n} \binom{n+\lfloor \frac{i}{2} \rfloor}{i} M^{4n} \left(M^2 + M^{-2} - \frac{1 + LM^{6+4n}}{M^2(1 + LM^{2+4n})} - 1 \right)^i \\
& \times \left(\frac{1 + LM^{6+4n}}{M^2(1 + LM^{2+4n})} \right)^{\lfloor \frac{1+i}{2} \rfloor}.
\end{aligned}$$

We observe that

$$M^2 + M^{-2} - \frac{1 + LM^{6+4n}}{M^2(1 + LM^{2+4n})} - 1 = \frac{(LM^{4n} - 1)(1 - M^2)}{(1 + LM^{2+4n})}.$$

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The resulting expression,

$$\sum_{i=0}^{2n} \binom{n+\lfloor \frac{i}{2} \rfloor}{i} M^{4n} \left(\frac{(LM^{4n} - 1)(1 - M^2)}{1 + LM^{2+4n}} \right)^i \left(\frac{1 + LM^{6+4n}}{M^2 + LM^{4+4n}} \right)^{\lfloor \frac{1+i}{2} \rfloor},$$

once denominators are cleared and some power of M is factored out to give a polynomial, gives the A-polynomial $A_{2n}(L, M)$. We multiply it by $M^{-6n}(1+LM^{2+4n})^{3n}$ so that we have the claimed formula in Theorem 1.1. The following equality, which guarantees that the claimed formula has the constant term 1 and is a polynomial,

$$c_{2n} = \sum_{i=0}^{2n} \binom{n+\lfloor \frac{i}{2} \rfloor}{i} (M^2 - 1)^i \left(\frac{1}{M^2} \right)^{\lfloor \frac{1+i}{2} \rfloor} M^{-2n} = 1,$$

can be proved by induction,

$$c_{2n} = \frac{(M^2 - 1)^2 c_{2(n-1)}}{M^4} + \frac{2c_{2(n-1)}}{M^2} - \frac{c_{2(n-2)}}{M^4},$$

which is proved in the following. As in the case of the proof of Lemma 2.2, c_{2n} can be regarded as an infinite sum. Direct computation shows that $f_2 = c_2$ and $f_4 = c_4$. Let f_{2n} be the right side of the claimed formula. We will show that $f_{2n} = c_{2n}$. Again as in the case of the proof of Lemma 2.2, f_{2n} can be written as

$$\sum_i \left[\binom{n-2+\lfloor \frac{i}{2} \rfloor}{i-2} + 2 \binom{n-1+\lfloor \frac{i}{2} \rfloor}{i} - \binom{n-2+\lfloor \frac{i}{2} \rfloor}{i} \right] \times (M^2 - 1)^i \left(\frac{1}{M^2} \right)^{\lfloor \frac{1+i}{2} \rfloor} M^{-2n}.$$

Hence as in the case of the proof of Lemma 2.2, by using the binomial relations three times, we have $f_{2n} = c_{2n}$.

Similarly, for $n < 0$, substituting

$$-\frac{1 + LM^{6+4n}}{M^2(1 + LM^{2+4n})} = -\frac{M^{-4n} + LM^6}{M^2(M^{-4n} + LM^2)}$$

for x into P_{2n} gives

$$\sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-2} \left(-M^2 - M^{-2} + \frac{M^{-4n} + LM^6}{M^2(M^{-4n} + LM^2)} + 1 \right)^i \times \left(\frac{M^{-4n} + LM^6}{M^2(M^{-4n} + LM^2)} \right)^{\lfloor \frac{1+i}{2} \rfloor}.$$

We observe that

$$-M^2 - M^{-2} + \frac{M^{-4n} + LM^6}{M^2(M^{-4n} + LM^2)} + 1 = \frac{(M^{-4n} - L)(1 - M^2)}{(M^{-4n} + LM^2)}.$$

The resulting expression,

$$\sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} M^{-4n-2} \left(\frac{(1-M^2)(M^{-4n}-L)}{LM^2+M^{-4n}} \right)^i \left(\frac{LM^6+M^{-4n}}{LM^4+M^{2-4n}} \right)^{\lfloor \frac{1+i}{2} \rfloor},$$

once denominators are cleared and some power of M is factored out to give a polynomial, gives the A-polynomial $A_{2n}(L, M)$. We multiply it by $M^{12n+8} (LM^2 + M^{-4n})^{-3n-1}$ so that we have the claimed formula in Theorem 1.1:

$$\begin{aligned} & \sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} \left(\frac{(1-M^2)(M^{-4n}-L)}{M^2(L+M^{-4n-2})} \right)^i \left(\frac{M^2(L+M^{-4n-6})}{L+M^{-4n-2}} \right)^{\lfloor \frac{1+i}{2} \rfloor} \\ & \times M^{2n+4} (L+M^{-4n-2})^{-3n-1}. \end{aligned}$$

Now we want to show that the claimed formula does not have fractions. For $n = -1$, by direct computation, one can show that the claimed formula does not have fractions. For each $n < -1$, fractions can only occur in the following sums.

$$\begin{aligned} & \sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} ((1-M^2)(-L))^i L^{\lfloor \frac{1+i}{2} \rfloor} \\ & \times M^{2n+4-2i+2\lfloor \frac{1+i}{2} \rfloor} L^{-3n-1-i-\lfloor \frac{1+i}{2} \rfloor} \\ & = \sum_{i=0}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} (M^2-1)^i M^{2n+4-2i+2\lfloor \frac{1+i}{2} \rfloor} L^{-3n-1}. \end{aligned}$$

Let c_{2n} be the coefficient of L^{-3n-1} of the above sum. Then, one can prove that $c_{2n} = M^4$ by the following recurrence relation,

$$c_{2n} = \frac{(M^2-1)^2 c_{2(n+1)}}{M^4} + \frac{2c_{2(n+1)}}{M^2} - \frac{c_{2(n+2)}}{M^4},$$

which can be proved as in the case of $n > 0$.

Now, we are going to compute a part of the coefficient of L^{-3n-2} . For each $n < 0$, the term $-L^{-3n-2}$ exists:

$$\begin{aligned} & \sum_{i=-2n-2}^{-2n-1} \binom{-n+\lfloor \frac{i-1}{2} \rfloor}{i} M^{2n+4-2i+2\lfloor \frac{1+i}{2} \rfloor} \\ & \times (-L)^i \left(\left\lfloor \frac{1+i}{2} \right\rfloor L^{\lfloor \frac{1+i}{2} \rfloor - 1} M^{-4n-6} \right) L^{-3n-1-i-\lfloor \frac{1+i}{2} \rfloor} \\ & = \sum_{i=-2n-2}^{-2n-1} \left\lfloor \frac{1+i}{2} \right\rfloor L^{-3n-2}. \end{aligned}$$

And now we are going to compute a part of the coefficient of L^0 . For each $n < 0$,

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the term $M^{12n^2+14n+6}$ exists:

$$\begin{aligned} & \sum_{i=-2n-1}^{-2n-1} \binom{-n + \lfloor \frac{i-1}{2} \rfloor}{i} M^{2n+4-2i+2\lfloor \frac{1+i}{2} \rfloor} \\ & \times (M^{-4n})^i (M^{-4n-6})^{\lfloor \frac{1+i}{2} \rfloor} (M^{-4n-2})^{-3n-1-i-\lfloor \frac{1+i}{2} \rfloor} \\ & = \sum_{i=-2n-1}^{-2n-1} M^{12n^2+12n+6-2\lfloor \frac{1+i}{2} \rfloor}. \end{aligned}$$

Hence there does not exist redundant L or M factors.

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